X <-

matrix (c(-0.0483954,11.57731,0.534487, -0.0133315,11.61102,0.532328, 0.0879925,11.61344,0.547736, 0.1619318,11.71156,0.540846, 0.1485665,12.18896,0.591167, 0.1602123,12.48978,0.575417, 0.2550375,12.48162,0.594495, 0.3297856,12.6648,0.597409, 0.4779284,12.85868,0.638522, 0.6018211,13.25208,0.676287, 0.4356969,13.67813,0.605735, 0.4238942,13.81275,0.61436, 0.5069381,13.75151,0.633366, 0.6001049,13.66419,0.650117, 0.6608616,13.62121,0.625603, -0.652706,11.55017,0.490851, -0.626186,11.62157,0.473449, -0.4228269,11.68405,0.503013, -0.2337306,11.65092,0.512501, -0.1708536,12.27989,0.566782, -0.1591224,12.54861,0.558133, -0.0802962,12.62747,0.558799, 0,12.76171,0.57207, 0.181029,12.83356,0.624763, 0.2931695,13.2069,0.628706, 0.2823478,13.65693,0.58915, 0.2219589,13.81871,0.532612, 0.2265936,13.75574,0.526652, 0.3161019,13.66065,0.540163, 0.3291167,13.61872,0.528775, -1.337794,11.6851,0.524334, -1.322632,11.72641,0.537, -1.18403,11.71898,0.582119, -1.122129,11.78504,0.579489, -1.062167,12.3106,0.606592, -1.000986,12.53797,0.60727, -0.8917518,12.63318,0.582425, -0.8029107,12.78289,0.573972, -0.6169364,12.84347,0.654256, -0.6173313,13.2272,0.631055, -0.7593575,13.65337,0.56924, -0.7972996,13.831,0.589682, -0.757594,13.76895,0.587953, -0.7044164,13.6956,0.565388, -0.7066033,13.646,0.577078, -2.448849,11.65257,0.432066, -2.335728,11.69941,0.439669, -1.955456,11.71102,0.488932, -1.773635,11.75367,0.484181, -1.749821,12.25203,0.529925, -1.806232,12.48047,0.532723,

```
-1.766641,12.66579,0.549067,
          -1.726871,12.8038,0.55714,
          -1.648969,12.87245,0.611377,
          -1.416881,13.21289,0.645319,
          -1.360607,13.65348,0.611734,
         -1.387667,13.82662,0.580884,
          -1.294908,13.76625,0.572047,
         -0.9912001,13.68918,0.59457,
          -0.8641466,13.63083,0.585525,
          -2.975381,11.68032,0.442875,
          -2.944165,11.66327,0.462473,
          -2.876209,11.66008,0.519118,
          -2.703764,11.77045,0.529331,
          -2.65487,12.40191,0.557797,
          -2.604217,12.53512,0.556181,
         -2.465739,12.66752,0.569327,
         -2.348901,12.7905,0.583465,
         -2.121815,12.89337,0.631818,
         -1.896813,13.24754,0.604723,
          -1.937845,13.6511,0.587921,
         -1.776083,13.82123,0.616159,
          -1.755846,13.77284,0.605868,
          -1.678413,13.6796,0.594688,
          -1.545154,13.64664,0.635545,
          -3.278573,11.67089,0.448539,
         -3.22429,11.6904,0.475889,
         -3.116071,11.6621,0.500562,
          -2.990844,11.71992,0.500344,
         -2.899586,12.17721,0.528897,
          -2.947666,12.6376,0.495361,
         -2.865108,12.68725,0.510342,
          -2.788881,12.80238,0.518296,
          -2.673258,12.86469,0.546723,
          -2.377858,13.24435,0.554276,
          -2.183558,13.68177,0.517766,
         -1.869803,13.82859,0.580049,
         -1.679533,13.74345,0.556024,
          -1.398987,13.65496,0.537791,
         -1.190685,13.61642,0.525775),
nrow=90,
byrow=TRUE,
dimnames=list (1:90))
matrix (c(13.9471,
         14.01082,
         14.08521,
         14.22863,
         14.33236,
         14.4164,
         14.52004,
         14.65482,
         14.78597,
         14.99343,
         15.14728,
```

Y <-

- 15.16818,
- 15.20081,
- 15.27014,
- 15.3733,
- 13.25215,
- 13.37018,
- 13.56404,
- 13.8148,
- 14.00113,
- 14.1216,
- 14.22188,
- 14.35158,
- 14.52128,
- 14.75096,
- 14.95901,
- 15.08463,
- 15.12863,
- 15.19235,
- 15.25283,
- 12.56479,
- 12.64203,
- 12.74273,
- 12.8336,
- 13.01709,
- 3 7 3
- 13.14297,
- 13.26273,
- 13.41403,
- 13.57191,
- 13.72546,
- 13.85619,
- 13.934,
- 13.90724,
- 13.99694,
- 13.97292,
- 11.88564,
- 12.04468,
- 12.41919,
- 12.64236,
- 12.77801,
- 12.83185,
- 12.95019,
- 13.069,
- 13.18551,
- 13.42509,
- 13.68818,
- 13.86622,
- 13.99255,
- 14.08048,
- 14.17805,
- 11.42257,
- 11.46613,
- 11.49463,
- 11.66106,
- 11.83777,

```
12.11816,
   12.25587,
   12.52097,
   12.78525,
   12.97698,
   13.16981,
   13.18237,
   13.27328,
   13.32164,
   11.14154,
   11.22396,
   11.33653,
   11.49423,
   11.68224,
   11.79931,
   11.88492,
   12.04773,
   12.20495,
   12.53104,
   12.85181,
   13.1362,
   13.35884,
   13.59784,
   13.82497),
nrow=90,
byrow=TRUE,
dimnames=list (1:90))
Z <-
1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,
    1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,
    1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,
```

11.95907,

0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0, 0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0, 0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0, 0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0, 0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0, 0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0, 0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0, 0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0, 0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0, 0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0, 0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0, 0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0, 0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0, nrow=90, byrow=TRUE, dimnames=list (1:90)) X1 <matrix (c(-0.0483954, -0.0133315, 0.0879925, 0.1619318, 0.1485665, 0.1602123, 0.2550375, 0.3297856, 0.4779284, 0.6018211, 0.4356969, 0.4238942, 0.5069381, 0.6001049, 0.6608616, -0.652706, -0.626186, -0.4228269, -0.2337306, -0.1708536, -0.1591224, -0.0802962, 0, 0.181029, 0.2931695, 0.2823478, 0.2219589, 0.2265936, 0.3161019, 0.3291167, -1.337794, -1.322632, -1.18403, -1.122129,

> -1.062167, -1.000986, -0.8917518, -0.8029107, -0.6169364,

- -0.6173313,
- -0.7593575,
- -0.7972996,
- -0.757594,
- -0.7044164,
- -0.7066033,
- -2.448849,
- -2.335728,
- -1.955456,
- -1.773635,
- -1.749821,
- -1.806232,
- -1.766641,
- -1.726871,
- -1.648969,
- -1.416881,
- -1.360607,
- -1.387667,
- -1.294908,
- -0.9912001,
- -0.8641466,
- -2.975381,
- -2.944165,
- -2.876209,
- -2.703764,
- -2.65487,
- -2.604217,
- -2.465739,
- -2.348901,
- -2.121815,
- -1.896813,
- -1.937845,
- -1.776083,
- -1.755846,
- -1.678413,
- -1.545154,
- -3.278573,
- -3.22429,
- -3.116071,
- -2.990844,
- -2.899586,
- -2.947666,
- -2.865108,
- -2.788881,
- -2.673258,
- -2.377858,
- -2.183558,
- -1.869803,
- -1.679533,
- -1.398987,
- -1.190685),

nrow=90,

byrow=TRUE,

dimnames=list (1:90))

```
X2 <-
matrix (c(11.57731,
          11.61102,
          11.61344,
          11.71156,
          12.18896,
          12.48978,
          12.48162,
          12.6648,
          12.85868,
          13.25208,
          13.67813,
          13.81275,
          13.75151,
          13.66419,
          13.62121,
          11.55017,
          11.62157,
          11.68405,
          11.65092,
          12.27989,
          12.54861,
          12.62747,
          12.76171,
          12.83356,
          13.2069,
          13.65693,
          13.81871,
          13.75574,
          13.66065,
          13.61872,
          11.6851,
          11.72641,
          11.71898,
          11.78504,
          12.3106,
          12.53797,
          12.63318,
          12.78289,
          12.84347,
          13.2272,
          13.65337,
          13.831,
          13.76895,
          13.6956,
          13.646,
          11.65257,
          11.69941,
          11.71102,
```

11.75367, 12.25203, 12.48047, 12.66579, 12.8038,

```
12.87245,
         13.21289,
         13.65348,
         13.82662,
         13.76625,
         13.68918,
          13.63083,
         11.68032,
         11.66327,
         11.66008,
         11.77045,
         12.40191,
         12.53512,
         12.66752,
         12.7905,
         12.89337,
         13.24754,
         13.6511,
         13.82123,
         13.77284,
         13.6796,
         13.64664,
          11.67089,
         11.6904,
         11.6621,
         11.71992,
         12.17721,
         12.6376,
         12.68725,
          12.80238,
         12.86469,
         13.24435,
         13.68177,
         13.82859,
         13.74345,
         13.65496,
         13.61642),
nrow=90,
byrow=TRUE,
dimnames=list (1:90))
matrix (c(0.534487,
         0.532328,
         0.547736,
         0.540846,
         0.591167,
         0.575417,
         0.594495,
         0.597409,
         0.638522,
         0.676287,
         0.605735,
          0.61436,
```

0.633366,

X3 <-

- 0.650117,
- 0.625603,
- 0.490851,
- 0.473449,
- 0.503013,
- 0.512501,
- 0.566782,
- 0.558133,
- 0.558799,
- 0.57207,
- 0.624763,
- 0.628706,
- 0.58915,
- 0.532612,
- 0.526652,
- 0.540163,
- 0.528775,
- 0.524334,
- 0.537,
- 0.582119,
- 0.579489,
- 0.606592,
- 0.60727,
- 0.582425,
- 0.573972,
- 0.654256,
- 0.631055,
- 0.56924,
- 0.589682,
- 0.587953,
- 0.565388,
- 0.577078,
- 0.432066,
- 0.439669,
- 0.488932,
- 0.484181,
- 0.529925,
- 0.532723,
- 0.549067,
- 0.55714,
- 0.611377,
- 0.645319,
- 0.611734,
- 0.580884,
- 0.572047,
- 0.59457,
- 0.585525,
- 0.442875,
- 0.462473,
- 0.519118,
- 0.529331,
- 0.557797,
- 0.556181,
- 0.569327,

```
0.583465,
         0.631818,
         0.604723,
         0.587921,
         0.616159,
         0.605868,
         0.594688,
         0.635545,
         0.448539,
         0.475889,
         0.500562,
         0.500344,
         0.528897,
         0.495361,
         0.510342,
         0.518296,
         0.546723,
         0.554276,
         0.517766,
         0.580049,
         0.556024,
         0.537791,
         0.525775),
nrow=90,
byrow=TRUE,
dimnames=list (1:90))
V \leftarrow cov(X)
p1=t(X)%*%Z
p2=t(p1)%*%solve(V)
p2
p3=t(p2)%*%t(Z)%*%X
p3
p4=solve(p3)
р4
p5=p4%*%t(p2)%*%t(Z)%*%Y
X1_hat = 0.7708356*X1
X1_hat
X2_hat = 1.0743491*X2
X2_hat
X3_{hat} = 0.9653712*X3
X3_hat
Y_hat = X1_hat + X2_hat + X3_hat
Y_hat
```

data var trans; input y x1 x2 x3; datalines; 1.695779 0.240937 0.067961 1.8967 0.058034 0.242867 1.872087 1.686131 1.802852 1.625124 0.239183 0.037641 0.029905 1.778645 1.568445 0.281789 0.042154 1.724236 1.51068 0.2365 1.707182 1.517337 0.274445 0.035509 1.699316 1.52391 0.179694 0.048194 1.698496 1.521251 0.225952 0.044955 0.328879 0.036526 1.667668 1.517451 1.644453 1.480897 0.361039 0.068948 1.592377 1.332542 0.367543 0.040422 1.470454 1.265804 0.345615 0.043818 1.401598 1.226351 0.342335 0.084614 1.421512 1.199393 0.34228 0.060817 1.091468 0.213797 0.024325 1.20175 1.281682 1.114304 0.309793 -0.00708 1.364815 1.172782 0.221149 0.00156 0.32743 0.017769 1.393006 1.19126 1.412382 1.198003 0.333275 0.018225 1.401156 1.188576 0.325544 0.012498 1.395256 1.191466 0.322862 0.019616 1.402978 1.220552 0.303759 0.022767 1.401983 1.172246 0.315859 0.021367 0.023837 1.179193 0.346343 1.404107 1.063869 0.351575 -0.03793 1.386904 0.998577 0.350333 -0.03872 1.364103 0.942348 0.338795 -0.02534 1.323808 0.33979 0.867649 -0.03601 1.301042 0.348727 0.51439 0.40638 0.057808 0.37683 0.503297 0.358257 0.062891 0.460372 0.353101 0.344723 0.072024 0.355269 0.383615 0.284384 0.068548 0.408966 0.299946 0.35814 0.057579 0.433752 0.356139 0.322635 0.067362 0.442006 0.377121 0.331254 0.036124 0.457706 0.388555 0.344042 0.021518 0.453608 0.422586 0.313669 0.05226 0.376483 0.261745 0.336159 0.023716 0.301287 0.137487 0.342783 0.003927 0.236274 0.04461 0.363865 0.01914 0.142713 0.01439 0.363543 0.022581 -0.07817 0.373745 -0.00012 0.128398 0.36707 0.021132 -0.16807 0.012292 -0.03446 -0.16476 -0.70467 0.316197 -0.09405 -0.63627 0.331257 -0.03462 -0.41832 -0.02116 0.136832 0.336763 -0.02676 0.192375 -0.36712 0.323899 0.169886 -0.38771 0.29957 -0.019090.265135 0.122632 -0.44911 -0.00718 0.002766 0.129466 -0.49777 0.363864 -0.53541 0.112676 0.364952 0.004686 0.067208 -0.60945 0.342649 0.009381 0.076113 -0.5378 0.321849 0.03798 -0.46376 0.342893 0.046421 0.133277 -0.54576 0.168494 0.359485 0.010342

0.360843

0.006675

-0.52292

0.211938 -0.36495 0.367325 0.029067

0.228023

```
0.226262 -0.32561
                     0.3519
                               0.020739
-0.62783
                               -0.02365
          -1.23121
                     0.343947
-0.6726
          -1.2447
                     0.295117
                               -0.01182
          -1.33908
-0.78773
                     0.285823
                               0.009023
-0.78892
          -1.29725
                     0.340679
                               0.01839
          -1.29276
                     0.44945
                               0.008784
-0.77035
          -1.24709
                     0.319785
                               0.016273
-0.75015
          -1.19687
-0.70256
                     0.365594
                             0.023026
-0.70045
          -1.15744
                    0.351652
                              0.031011
-0.59733
        -1.08229
                    0.363569
                              0.029822
        -1.01774
-0.56373
                    0.356499
                               -0.00262
-0.57792
        -1.041
                    0.340513 0.022608
-0.52792 \quad -0.93417
                   0.354095 0.045617
-0.58216 -0.98386
                             0.040496
                  0.367433
-0.59526 -1.05217
                   0.357745
                              0.029185
-0.63015 -1.00662
                              0.070759
                  0.36771
-0.90886 -1.5344
                    0.334517
                               -0.01799
-0.91477 -1.52483 0.322247 0.001595
-0.94583 -1.57894 0.287843 -0.00953
-0.95575 -1.58433 0.290149 -0.0106
-0.92588 -1.53747 0.22475
                              -0.02012
-0.90991 -1.59054 0.422265 -0.04455
-0.9358
         -1.59624
                   0.385324
                             -0.03596
-0.90859
        -1.59742
                   0.363532
                              -0.03416
-0.91335
        -1.63374
                   0.334889
                             -0.05527
                    0.353309
                               -0.05306
-0.81794
          -1.49878
                     0.371183
-0.70309
          -1.28671
                               -0.04755
                    0.361455
-0.56153
          -1.02789
                               0.009507
          -0.90755
-0.40569
                    0.338043
                               -0.00935
          -0.77274
-0.2707
                    0.333105
                               -0.02771
-0.12682 -0.65215 0.33749 -0.03901
proc reg data=var trans ;
model y=x1 x2 x3/noint;
run;
```