

ABSTRAK

KONSEP NORM PADA RING $\mathbb{Z}[i]$ DAN APLIKASINYA PADA PENYELESAIAN PERSAMAAN DIOPHANTINE NON LINEAR TIGA VARIABEL

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Himpunan bilangan bulat Gaussian merupakan himpunan bagian dari himpunan bilangan kompleks yang dinotasikan dengan $\mathbb{Z}[i]$ dimana $\mathbb{Z}[i] = \{a + bi \mid a, b \in \mathbb{Z}\}$. Di $\mathbb{Z}[i]$, suatu ukuran baik panjang maupun jarak dihitung dengan menggunakan norm. Norm pada $\mathbb{Z}[i]$ merupakan suatu fungsi $N : \mathbb{Z}[i] \rightarrow \mathbb{Z}$. Salah satu penggunaan konsep norm pada ring $\mathbb{Z}[i]$ yaitu dapat menyelesaikan persamaan Diophantine non linear. Pada penelitian ini, konsep norm pada ring $\mathbb{Z}[i]$ digunakan pada persamaan Diophantine non linear tiga variabel untuk mengklasifikasikan solusi primitif persamaan triple Pythagoras dan persamaan $a^2 + b^2 = c^3$. Hasilnya menunjukkan bahwa diperoleh solusi primitif untuk persamaan triple Pythagoras $a^2 + b^2 = c^2$ dengan a ganjil memiliki bentuk $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$ dimana $m > n > 0$, $\text{fpb}(m, n) = 1$ dan $m \not\equiv n \pmod{2}$. Kemudian untuk persamaan $a^2 + b^2 = c^3$ dengan $\text{fpb}(a, b) = 1$ diperoleh solusi primitif $a = m^3 - 2mn^2$, $b = 3m^2n - n^3$, $c = m^2 + n^2$, dimana $m > n > 0$, $\text{fpb}(m, n) = 1$ dan $m \not\equiv n \pmod{2}$.

Kata Kunci: *norm, prima, bilangan bulat Gaussian, ring $\mathbb{Z}[i]$, persamaan Diophantine*

ABSTRACT

THE CONCEPT OF NORM ON RING $\mathbb{Z}[i]$ AND ITS APPLICATION ON THREE VARIABLES NONLINEAR DIOPHANTINE EQUATIONS

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Gaussian integers are the subset of complex numbers, denoted by $\mathbb{Z}[i]$ and defined by $\mathbb{Z}[i] = \{ a + bi \mid a, b \in \mathbb{Z} \}$. In $\mathbb{Z}[i]$, size is measured by the norm. The function $N : \mathbb{Z}[i] \rightarrow \mathbb{Z}$, called the norm. Application of norm on ring $\mathbb{Z}[i]$ concept can be used to solve nonlinear Diophantine equations by applying the concept of norm on ring $\mathbb{Z}[i]$ into three variables nonlinear Diophantine equations. The applications will address the following issues: classification of (primitive) Pythagorean triples and classification of (primitive) solutions to $a^2 + b^2 = c^3$. The solutions are every primitive Pythagorean triple (a, b, c) with a is an odd number, and has the form $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$ where $m > n > 0$, $\gcd(m, n) = 1$ and $m \not\equiv n \pmod{2}$, and the solutions to $a^2 + b^2 = c^3$ with $\gcd(a, b) = 1$ are described by the formula $a = m^3 - 2mn^2$, $b = 3m^2n - n^3$, $c = m^2 + n^2$ where $m > n > 0$, $\gcd(m, n) = 1$ and $m \not\equiv n \pmod{2}$.

Keywords: *norm, prime, Gaussian integers, ring $\mathbb{Z}[i]$, Diophantine equations.*