

**APPLICATION OF PANEL VECTOR AUTOREGRESSIVE (PVAR)  
MODEL ON THE ANALYSIS OF INFLATION AND GRDP  
GROWTH RATE OF PROVINCES IN INDONESIA**

**(Thesis)**

**By**

***Khairunnisa***



**FACULTY OF MATHEMATICS AND NATURAL SCIENCES  
LAMPUNG UNIVERSITY  
BANDAR LAMPUNG  
2025**

## **ABSTRACT**

### **APPLICATION OF PANEL VECTOR AUTOREGRESSIVE (PVAR) MODEL ON THE ANALYSIS OF INFLATION AND GRDP GROWTH RATE OF PROVINCES IN INDONESIA**

**By**

**KHAIRUNNISA**

PVAR is an extension of the VAR model applied to panel data, which combines time series data with cross-section data from various regions. This model allows all variables to influence each other and be analyzed simultaneously as endogenous variables. This study aims to analyze the relationship between inflation and economic growth (GRDP) across provinces in Indonesia using the Panel Vector Autoregressive (PVAR) model. The analysis involves stationarity testing (IPS test), determining the optimal lag length (MMSC), and parameter estimation using the Generalized Method of Moments (GMM). Instrument validity is tested using the Sargan-Hansen test, and causal relationships are examined through the Granger causality test. The findings reveal a bidirectional relationship between inflation and economic growth in several provinces. The constructed model has also been proven to be stable. The Impulse Response Function (IRF) and Forecast Error Variance Decomposition (FEVD) analyses demonstrate how changes in one variable affect the other over time. These results are expected to serve as a valuable reference for formulating more effective regional economic policies.

**Keywords:** PVAR, GRDP, Inflation, Panel Data, GMM, IRF, FEVD

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**By**

***Khairunnisa***

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**FACULTY OF MATHEMATICS AND NATURAL SCIENCES  
LAMPUNG UNIVERSITY  
BANDAR LAMPUNG  
2025**

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ON THE ANALYSIS OF INFLATION AND  
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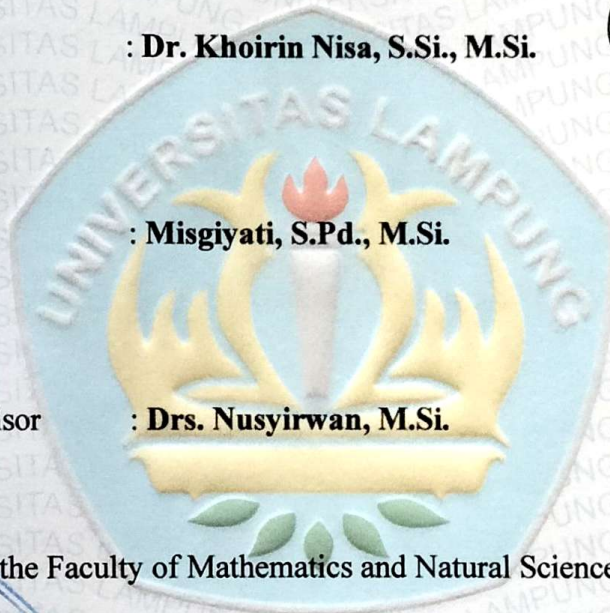
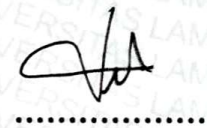
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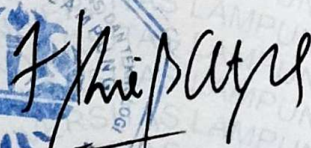
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## **BIOGRAPHY**

The author, whose full name is Khairunnisa, was born in Pasar Baru, Pesawaran, on August 15, 2003. She is the eldest child of Mr. Sihabuddin and Mrs. Dewi Andayani, and the older sister of Sultan Ghamar Syah and Muhammad Alwi Sihab.

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## **WORDS OF INSPIRATION**

*“Allah does not burden a soul beyond that it can bear”*

**(QS. Al-Baqarah: 286)**

*“And say, ‘My Lord, increase me in knowledge’”*

**(QS. Taha: 114)**

*“Knowledge is not only found in books, but also in action”*

**(Albert Einstein)**

## **DEDICATION**

With heartfelt gratitude to Allah SWT, for His grace and blessings that have enabled me to complete this thesis, I dedicate this work to:

My beloved parents, Sihabuddin and Dewi Andayani, and my dearest grandmother, Surtiasih, whose patience, sincerity, and unwavering support both moral and material have accompanied me throughout my educational journey. Thank you for your endless love and sacrifices, which have shaped the person I am today.

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Bandar Lampung, April 21, 2025  
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## **I. INTRODUCTION**

### **1.1. Background and Problem Statement**

In the complexity of today's global economic system, two primary concepts that remain the focus of discussion and analysis are inflation and economic growth (Saefulloh & Fahlevi, 2023). Developing countries generally face economic challenges such as high inflation and slow growth. Economic growth is defined as a country's long-term ability to produce an expanding range of goods for its growing population (Simanungkalit, 2020). When a government fails to boost its economic growth, new economic and social issues may arise. Growth is typically measured by Gross Domestic Regional Product (GRDP) (Salim et al., 2021). Inflation is a critical economic indicator, with efforts made to keep its rate low and stable to prevent macroeconomic problems. High and unstable inflation reflects economic instability, leading to a continuous rise in the prices of goods and services and, ultimately, an increase in poverty levels within the country.

A time series is a collection of observational values recorded over some time, usually with equally long intervals (Febrianti et al., 2021). Time series data can be recorded in various periods such as daily, weekly, monthly, annually, or other consistent periods (Cryer & Chan, 2008). This data analysis can be categorized based on the number of variables observed. If the data includes only one variable, it is called a univariate time series. Analysis with two variables is known as a bivariate time series, which provides better results when additional variables are involved. Furthermore, multivariate time series are used to analyze several interrelated variables in a system. Thus, observations of one variable are not only

influenced by previous observations of that variable, but also by observations of other variables. One model that can be used for quantitative analysis of multivariate time series is the Vector Autoregressive (VAR). In general, time series data related to the economy tend to be non-stationary. Therefore, it is necessary to perform differentiation two to three times, and so on, until stationary data is obtained. The method used in this stationarity test is the unit root test developed by Dickey-Fuller. Unit root testing aims to determine whether there is a unit root in the data. If the data contains unit roots, then the data is considered non-stationary (Hendajany & Wati, 2020).

The VAR model is a development of the Autoregressive (AR) model. In the AR model, there is only one variable being analyzed, while in the VAR model, there are many variables that are all considered endogenous variables. Endogenous variables are variables whose values are affected by other variables in the model, also known as dependent variables or dependent variables (Gujarati, 2009). Modeling using VAR only focuses on time series data from a single individual. However, in economic research, there are often data limitations that cannot represent the variables used in the analysis. These limitations can be an insufficient number of time units in time series data or a limited number of individuals in cross-section data. These data limitations can affect the accuracy of the two-way relationship analysis results in economic research. Therefore, combining both types of data, namely time series, and cross-section, into panel data can be done to analyze the relationship. The use of panel data has the advantage of explicitly accommodating heterogeneity among individuals.

Panel data analysis using the VAR method is called Panel Vector Autoregressive (PVAR). The PVAR model outperforms the traditional VAR technique by eliminating serious biases in the estimation. The PVAR model has clear advantages over covariance, which makes it possible to treat all variables considered in the regression as endogenous with panel data procedures, allowing for unobserved individual heterogeneity (Charfeddine & Kahia, 2019). PVAR imposes all variables as endogenous and checks for unobserved heterogeneity (Comunale, 2022) in both

a dynamic and static sense, although in some relevant cases, exogenous variables can be included (Canova & Ciccarelli, 2013). Many methods are unable to address endogeneity, which results in biased estimates and inconsistent estimators. To overcome these issues, the Generalized Method of Moments (GMM) is employed (Ouyang & Li, 2018). The PVAR model itself uses GMM estimation to determine its parameters (Hayakawa, 2016). Unlike average-based estimators, PVAR employs variance decomposition and impulse response functions to observe variable behavior over time. Consequently, detailed trends are revealed, proving useful for policy decision-making.

Previous research analyzed the causal relationship between inflation, Gross Regional Domestic Product (GRDP), Bank Indonesia's interest rate, the rupiah exchange rate against the US dollar, and global tin prices in Bangka Belitung Province using the Vector Auto Regression (VAR) and Vector Error Correction Model (VECM). The study by Sulistiana et al. (2017) demonstrated a long-term causal relationship among the examined variables, while in the short term, inflation affected Bank Indonesia's interest rate and GRDP was linked to other macroeconomic variables. Sihombing et al. (2022) examined the implementation of the Panel Vector Auto Regression (PVAR) model to study the relationship between inflation and the growth of the money supply in six ASEAN countries from 1994 to 2020. Their findings revealed a bidirectional causal relationship: an increase in inflation would decrease the growth of the money supply, whereas an increase in the money supply would, in turn, elevate inflation. Furthermore, Ouyang and Li (2018) and Usman et al. (2022) conducted research using the PVAR-GMM model on energy data, investigating renewable energy, financial development, and greenhouse gas emissions. Their results indicated a significant relationship between renewable energy and greenhouse gas emissions.

In this study, PVAR modeling will be applied to analyze economic growth and inflation in Indonesia using panel data. This study uses individual units consisting of 34 provinces in Indonesia, while the time units analyzed cover the period from 2015 to 2024. Then the causal relationship between time series variables will be

seen using Granger causality. To see the effect of a variable shock on other variables (see the impact of exogenous changes in each endogenous variable with other variables) in the PVAR system is done by estimating the Impulse Response Function (IRF) and Forecast Error Variance Decomposition (FEVD). FEVD is used to see how changes in a variable, as indicated by changes in error variance, are affected by other variables.

## **1.2. Research Objectives**

The objectives of this research is developing a Panel Vector Autoregressive (PVAR) model and determining the direction of the relationship between the variables in the panel data concerning Indonesia's GRDP growth rate and inflation rate during the period 2015 to 2024.

## **1.3. Research Benefits**

This research is expected to offer benefits by providing recommendations or considerations for the government in its efforts to promote economic growth and control inflation, thereby ensuring that the policies implemented are more precisely targeted. In addition, this study aims to supply scientific information and insights to readers regarding the application of tests for the directional relationship between two-panel variables and the modeling of the Panel Vector Autoregressive.

## II. LITERATURE REVIEW

### 2.1. Matrix

A matrix is a collection of numbers arranged in a specific structure of rows and columns to form a square or rectangular shape, written between two brackets, either  $()$  or  $[\ ]$ . The numbers arranged within the matrix are referred to as the elements or components of the matrix. The number of rows  $\times$  the number of columns determines the size of the matrix, which is known as the order of the matrix. A matrix can be represented in the following form:

$$A_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn} \end{bmatrix}$$

A matrix can also be expressed as  $A_{m \times n} = [a_{ij}]_{m \times n}$ , where  $a_{ij}$  represents the elements or entries of the matrix,  $i = (1, 2, \dots, m)$  denotes the row indices, and  $j = (1, 2, \dots, n)$  denotes the column indices.

#### 2.1.1. Types of Matrices

The types of matrices based on their elements are as follows:

1. Square matrix is a matrix in which the number of rows is equal to the number of columns.

2. Symmetric matrix is a square matrix in which the element  $a_{ij}$  is equal to the element  $a_{ji}$  or  $(a_{ij} = a_{ji})$  for all  $i$  and  $j$ .
3. Diagonal matrix is a matrix in which all elements outside the main diagonal are zero (0), and at least one element on the main diagonal is non-zero.
4. Identity matrix is a matrix in which all elements on the main diagonal are equal to one (1), and all elements outside the main diagonal are zero.
5. Scalar matrix is a matrix in which all elements on the main diagonal are equal to the same non-zero, non-one value, and all off-diagonal elements are zero.
6. Upper triangular matrix is a diagonal matrix in which some elements to the right of the main diagonal are non-zero.
7. Lower triangular matrix is a diagonal matrix in which some elements to the left of the main diagonal are non-zero.
8. Singular matrix is a matrix whose determinant is equal to zero.
9. Non-singular matrix is a matrix whose determinant is not equal to zero.
10. Elementary matrix is a square matrix of order  $n \times n$  obtained by performing a single elementary row operation on the identity matrix  $I_n$ .

### 2.1.2. Matrix Operations

The following are operations commonly used with matrices:

1. Matrix addition and subtraction

Let  $A = (a_{ij})$  and  $B = (b_{ij})$  be two matrices of the same size  $m \times n$ . The sum of matrices  $A$  and  $B$ , denoted  $A + B$  is an  $m \times n$  matrix in which each element is the sum of the corresponding elements from both matrices. The same rule applies to matrix subtraction. These operations are written as follows:

$$A + B = (a_{ij} + b_{ij})$$

$$A - B = (a_{ij} - b_{ij})$$

2. Scalar multiplication with a matrix

Let matrix  $A$  be given, and let  $c$  be a scalar. The matrix  $cA$  is obtained by multiplying each element of matrix  $A$  by  $c$ . This operation is written as follows:

$$c\mathbf{A} = (ca_{ij})$$

### 3. Matrix multiplication

If  $\mathbf{A} = (a_{ij})$  is an  $m \times n$  and  $\mathbf{B} = (b_{ij})$  is an  $n \times r$  matrix, then the product  $\mathbf{AB} = \mathbf{C} = (c_{ij})$  is an  $m \times r$  matrix whose entries are defined by:

$$c_{ij} = \sum_{k=1}^n a_{ik}b_{kj}$$

Properties of matrix multiplication:

- a.  $\mathbf{A}(\mathbf{BC}) = (\mathbf{AB})\mathbf{C}$
- b.  $\mathbf{A}(\mathbf{B} + \mathbf{C}) = \mathbf{AB} + \mathbf{AC}$
- c.  $(\mathbf{B} + \mathbf{C})\mathbf{A} = \mathbf{BA} + \mathbf{CA}$
- d.  $\mathbf{A}(\mathbf{B} - \mathbf{C}) = \mathbf{AB} - \mathbf{AC}$
- e.  $(\mathbf{B} - \mathbf{C})\mathbf{A} = \mathbf{BA} - \mathbf{CA}$
- f.  $a(\mathbf{BC}) = (ab)\mathbf{C} = \mathbf{B}(a\mathbf{C})$
- g.  $\mathbf{AI} = \mathbf{IA} = \mathbf{A}$

### 2.1.3. Matrix Transpose

If  $\mathbf{A}$  is a matrix of size  $m \times n$ , then the transpose of  $\mathbf{A}$ , denoted by  $\mathbf{A}^T$ , is defined as a matrix of size  $n \times m$  obtained by interchanging the rows and columns of matrix  $\mathbf{A}$ .

If matrix  $\mathbf{A}$  is expressed as:

$$\mathbf{A}_{m \times n} = (a_{ij}),$$

then the transpose of matrix  $\mathbf{A}$  expressed as:

$$\mathbf{A}^T = (b_{ji})$$

where  $b_{ji} = a_{ij}$ .

The main properties of the matrix transpose operation are as follows::

1.  $((\mathbf{A})^T)^T = \mathbf{A}$
2.  $(\mathbf{A} + \mathbf{B})^T = \mathbf{A}^T + \mathbf{B}^T$  and  $(\mathbf{A} - \mathbf{B})^T = \mathbf{A}^T - \mathbf{B}^T$
3.  $(k\mathbf{A})^T = k\mathbf{A}^T$  where  $k$  is any scalar.
4.  $(\mathbf{AB})^T = \mathbf{B}^T\mathbf{A}^T$

#### 2.1.4. Matrix Determinant

The determinant of an  $n \times n$  matrix, denoted as  $\det(\mathbf{A})$ , is a scalar associated with matrix  $\mathbf{A}$  and is defined as follows:

$$\det(\mathbf{A}) \begin{cases} a_{11}, & \text{if } n = 1 \\ a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}, & \text{if } n > 1 \end{cases}$$

where

$$A_{ij} = (-1)^{1+j} \det(M_{ij}), \quad j = 1, \dots, n$$

are the cofactors associated with the entries in the first row of matrix  $\mathbf{A}$ , and  $M_{ij}$  is the minor obtained by removing the first row and the  $j$ -th column of  $\mathbf{A}$ .

#### 2.1.5. Matrix Inverse

If  $\mathbf{A}$  is a square matrix and there exists a matrix  $\mathbf{B}$  of the same size such that  $\mathbf{AB} = \mathbf{BA} = \mathbf{I}$ , then  $\mathbf{A}$  is said to be invertible, and  $\mathbf{B}$  is called the inverse of  $\mathbf{A}$ . A matrix  $\mathbf{B}_{n \times n}$  is the inverse of a matrix  $\mathbf{A}_{n \times n}$  if it satisfies  $\mathbf{AB} = \mathbf{BA} = \mathbf{I}_n$ . The inverse of matrix  $\mathbf{A}$  is given by the formula:

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \text{adj}(\mathbf{A}).$$

### 2.2. Panel Data and Stationarity

Panel data is a type of data that encompasses observations from several individuals tracked over a specific period (Hsiao, 2014). Panel data has two dimensions: the individual (cross-section) dimension, denoted by  $i$ , and the time series dimension, denoted by  $t$ , where each unit is observed or recorded over multiple periods. The objective of using panel data methods is to generate more accurate estimates by increasing the number of observations, thereby enhancing the degrees of freedom. There are several types of panel data: balanced panels, unbalanced panels, short

panels, and long panels. Panel data is considered balanced if each subject has the same number of observations; conversely, if the number of observations differs among subjects, it is referred to as unbalanced. A short panel is characterized by having more individuals ( $N$ ) than periods ( $T$ ), whereas a long panel has fewer individuals ( $N$ ) than periods ( $T$ ).

Stationarity is a condition in which data does not experience significant changes or fluctuations around a constant mean, and is independent of time or variance. In other words, stationarity means that there is neither an increase nor a decrease in the observed values over time. The use of panel data results in a larger sample size, which can lead to changes in the data structure and an increased likelihood of heterogeneity, especially as the number of cross-section units grows. Therefore, it is important to conduct stationarity tests on panel data, which should be carried out for each cross-section unit. A time series is considered stationary if its process behavior remains consistent over time, in other words, if the process is in equilibrium (Cryer & Chan, 2008). There are several tests to assess the stationarity of panel data, one of which is the Im, Pesaran, and Shin (IPS) test (Hsiao, 2014). In the IPS test, the hypothesis  $H_0$  states that the data is not stationary or has a unit root. If the data is not stationary, then a differencing process is performed, where the original series is replaced with a difference series to make it stationary (Cryer & Chan, 2008).

Im, Pesaran, and Shin apply an alternative testing procedure that is based on the average test statistic results of each unit root test. To calculate the test statistics on the IPS test, it is first necessary to calculate the Augmented Dickey-Fuller (ADF) test statistics on the time series data in each unit cross-section ( $t_{pi}$ ) with  $i = 1, 2, \dots, N$  and  $N$  is the number of unit cross-sections shown in the following AR(1) model.

$$Y_t = \phi Y_{t-1} + e_t, \quad t = 1, 2, \dots, T \quad (2.1)$$

where the estimation of  $\phi$  for each cross-section unit using OLS is as follows.

$$\hat{\phi}_i = \frac{\sum_{t=1}^n Y_{i,t-1} Y_{i,t}}{\sum_{t=1}^n Y_{i,t-1}^2} \quad (2.2)$$

The hypothesis used in the IPS test is as follows (Baltagi, 2021):

$H_0: \phi_i = 1$  with  $i = 1, 2, \dots, N$  (data is not stationary).

$H_1: \phi_i < 1$  with  $i = 1, 2, \dots, N$  (stationary data).

The ADF test statistic for each cross-section unit  $i$ , where  $i = 1, 2, \dots, N$ , can be computed using the following formula.

$$t_{pi} = \frac{\hat{\phi}_i - 1}{S_{\hat{\phi}_i}} = \frac{\hat{\phi}_i - 1}{\left[ \sigma_{e_i}^2 (\sum_{t=1}^n Y_{i,t-1}^2)^{-1} \right]^{\frac{1}{2}}} \quad (2.3)$$

with

$$S_{\hat{\phi}_i} = \sqrt{\frac{\sigma_{e_i}^2}{\sum_{t=1}^n Y_{i,t-1}^2}} \quad \text{and} \quad \sigma_{e_i}^2 = \sum_{t=1}^n \frac{(Y_{i,t} - \hat{\phi}_i Y_{i,t-1})^2}{n-1}$$

After obtaining the ADF test statistic ( $t_{pi}$ ) for each cross section unit, the IPS test statistic can be calculated using the following formula.

$$\bar{t} = \frac{1}{N} \sum_{i=1}^N t_{pi} \quad (2.4)$$

with  $t_{pi}$  is the ADF test statistic at each cross-section unit.

The decision criterion is to reject  $H_0$  if  $\bar{t}$  is less than the critical value from the Dickey-Fuller table corresponding to the chosen significance level ( $\alpha$ ) in such a case, the panel data can be considered stationary.

### 2.3. Differencing

Differencing occurs when the panel data is non-stationary, meaning the original time series is replaced with its differenced series. For each cross-section unit, the initial form of differencing is as follows.

$$\Delta Y_{i,t} = Y_{i,t} - Y_{i,t-1} \quad (2.5)$$

where

$\Delta Y_{i,t}$  : first difference variable for the  $i$ -th individual at time  $t$ .

$Y_{i,t}$  : variable for the  $i$ -th individual at time  $t$ .

$Y_{i,t-1}$  : variable for the  $i$ -th individual at time  $(t - 1)$ .

If the first differencing has not produced stationary time series data for each cross-section, then a second differencing is necessary. The form of the second differencing is as follows.

$$\Delta^2 Y_{i,t} = \Delta Y_{i,t} - \Delta Y_{i,t-1} \quad (2.6)$$

where

$\Delta^2 Y_{i,t}$  : second-differenced variable for the  $i$ -th individual at time  $t$ .

$\Delta Y_{i,t}$  : variable for the  $i$ -th individual at time  $t$ .

$\Delta Y_{i,t-1}$  : variable for the  $i$ -th individual at time  $(t - 1)$ .

This is also true for subsequent differencing. However, in practice, it is rare to do more than two differentiations, as the original data generally becomes stationary with only one or two levels of differentiation.

## 2.4. Vector Autoregressive (VAR) Model

The Vector Autoregressive (VAR) model is an extension of the Autoregressive (AR) model from univariate time series to a framework that incorporates multiple variables. The VAR model is one of the approaches used for forecasting and is closely related to economics, including the formulation of macroeconomic policies. Generally, the VAR model aims to explain the dynamic behavior among interacting variables, which is further elaborated through functions such as Impulse Response and Variance Decomposition (Suyanto, 2023). In the VAR model, all variables are treated as endogenous and are interrelated both dynamically and statically, although in some cases, relevant exogenous variables may also be included. This model explains the interrelationships among the observations of each variable at previous periods as well as the interrelationships with observations of other variables from previous periods.

The VAR time series model represents several AR processes in matrix form. For example, a VAR model for two variables (bivariate) can be described as follows.

$$\begin{aligned} Y_{1t} &= c_1 + a_{11}y_{1,t-1} + a_{12}y_{2,t-1} + \varepsilon_{1t} \\ Y_{2t} &= c_2 + a_{21}y_{1,t-1} + a_{22}y_{2,t-1} + \varepsilon_{2t} \end{aligned} \quad (2.7)$$

In matrix notation, it can be written as follows.

$$\begin{bmatrix} Y_{1,t} \\ Y_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \quad (2.8)$$

The general form obtained is as follows.

$$\mathbf{Y}_t = \mathbf{c} + \mathbf{A}_1 \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \quad (2.9)$$

In general, a VAR model of order  $p$  can be formulated as follows (Febrianti et al., 2021).

$$\mathbf{Y}_t = \mathbf{c} + \mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_2 \mathbf{y}_{t-2} + \mathbf{A}_3 \mathbf{y}_{t-3} + \cdots + \mathbf{A}_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t \quad (2.10)$$

It can also be written as follows.

$$\mathbf{Y}_t = \mathbf{c} + \sum_{i=1}^p \mathbf{A}_i \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t \quad ; p > 0 \quad (2.11)$$

where

$\mathbf{Y}_t$  : an  $n \times 1$  vector, where  $n$  is the number of endogenous variables at time  $t$  and  $t - i$  ; for  $i = 1, 2, \dots, p$ .

$\mathbf{c}$  : a constant vector of size  $n \times 1$ .

$\mathbf{A}_i$  : a coefficient matrix of size  $n \times n$  or the endogenous variables.

$\boldsymbol{\varepsilon}_t$  : a random error vector of size  $n \times 1$ .

## 2.5. Panel Vector Autoregressive (PVAR) Model

The Panel Vector Autoregressive (PVAR) model is an extension of the Vector Autoregressive (VAR) model, where the data used consists of panel data a combination of cross-section and time series data. In the PVAR model, all variables are treated as endogenous and are interrelated. According to Holtz-Eakin et al. (1988), the general form of a system of linear equations for  $k$  PVAR variables with lag order  $p$  and fixed effects is as follows.

$$\mathbf{y}_{it} = \mathbf{A}_1 \mathbf{y}_{it-1} + \cdots + \mathbf{A}_p \mathbf{y}_{it-p} + \boldsymbol{\eta}_i + \mathbf{e}_{it} \quad (2.12)$$

$$i \in \{1, 2, \dots, N\}, t \in \{1, 2, \dots, T\}$$

where

$\mathbf{y}_{it}$  : an  $m \times 1$  vector of endogenous variables for cross-section unit  $i$  at period  $t$ .

$\mathbf{y}_{it-l}$  : an  $m \times 1$  vector of endogenous variables for cross-section unit  $i$  at period  $t - l$  where  $l = 0, \dots, p$ .

$\mathbf{e}_{it}$  : an  $m \times 1$  error vector for cross-section unit  $i$  at period.

$\mathbf{A}_l$  : an  $m \times m$  matrix of model parameters to be estimated, where  $l = 1, \dots, p$ .

$\boldsymbol{\eta}_i$  : an  $m \times 1$  fixed effects vector.

Equation (2.12) can be explained as follow.

$$\mathbf{y}_{it-l} = \begin{bmatrix} y_{it-l}^{(1)} \\ y_{it-l}^{(2)} \\ \vdots \\ y_{it-l}^{(k)} \end{bmatrix}_{k \times 1}, \text{ with } l = 1, \dots, p \quad \mathbf{e}_{it} = \begin{bmatrix} e_{it}^{(1)} \\ e_{it}^{(2)} \\ \vdots \\ e_{it}^{(k)} \end{bmatrix}_{k \times 1}$$

Equation (2.12) can be written in matrix form as follows.

$$\begin{bmatrix} y_{it}^{(1)} \\ y_{it}^{(2)} \\ \vdots \\ y_{it}^{(k)} \end{bmatrix} = \begin{bmatrix} a_{1,11} & a_{1,12} & \cdots & a_{1,1k} \\ a_{1,21} & a_{1,22} & \cdots & a_{1,2k} \\ \vdots & \vdots & \vdots & \vdots \\ a_{1,k1} & a_{1,k2} & \cdots & a_{1,kk} \end{bmatrix} \begin{bmatrix} y_{it-l}^{(1)} \\ y_{it-l}^{(2)} \\ \vdots \\ y_{it-l}^{(k)} \end{bmatrix} + \cdots \quad (2.13)$$

$$+ \begin{bmatrix} a_{p,11} & a_{p,12} & \cdots & a_{p,1k} \\ a_{p,21} & a_{p,22} & \cdots & a_{p,2k} \\ \vdots & \vdots & \vdots & \vdots \\ a_{p,k1} & a_{p,k2} & \cdots & a_{p,kk} \end{bmatrix} \begin{bmatrix} y_{it-l}^{(1)} \\ y_{it-l}^{(2)} \\ \vdots \\ y_{it-l}^{(k)} \end{bmatrix} + \begin{bmatrix} \eta_i^{(1)} \\ \eta_i^{(2)} \\ \vdots \\ \eta_i^{(k)} \end{bmatrix} + \begin{bmatrix} e_{it}^{(1)} \\ e_{it}^{(2)} \\ \vdots \\ e_{it}^{(k)} \end{bmatrix}$$

These parameters can be estimated using a fixed-effects approach after applying some transformations via the OLS method. However, since there is a lagged dependent variable on the right-hand side, the equation will yield biased and inconsistent estimates even with a large  $N$  and there will be a correlation between the lagged dependent variable on the right-hand side and the error term. This is known as the endogeneity problem. The endogeneity issue can be resolved by employing the Generalized Method of Moments (GMM) for parameter estimation,

standardizing the model based on the GMM system using lags rather than regressions as instruments. PVAR modeling can be summarized in three stages: model order identification, parameter estimation using GMM, and several tests to be conducted.

## 2.6. Parameter Estimation Generalized Method of Moment (GMM)

The generalized Method of Moments (GMM) is an extension of the method of moments, which is one approach to obtaining a consistent estimator for a parameter (Arsa et al., 2017). GMM was first introduced by Hansen in 1982 as a parameter estimation method based on moment conditions. This method can be applied to address violations of data assumptions, such as heteroskedasticity or autocorrelation. In the method of moments, the number of instrumental variables must be commensurate with the number of parameters to be estimated. However, if the number of instrumental variables exceeds the number of parameters, the method of moments cannot be applied. Therefore, GMM is utilized. The basic principle of GMM is to minimize the moment restriction, which is,  $E[\mathbf{y}_{it-1}, e] = 0$  (Hayakawa, 2016).

In identifying the parameters in equation (2.12), we must first observe the influence of individual effects. It should be noted that if the model is estimated using individual effects as well as variables containing lag elements, it will produce inconsistent estimates. Therefore, to overcome this problem, a transformation that removes the influence of individual effects is required. In this context, removing individual effects ( $\eta_i$ ) in the PVAR model will make the model consistent or uniform across regions. Parameter estimation is performed after the individual effect coefficients and lag coefficients reach stationary conditions. Equation (2.12) can then be transformed into the following model.

$$\begin{aligned} \mathbf{y}_{it}^* &= \mathbf{A}_1 \mathbf{y}_{i,t-1}^* + \cdots + \mathbf{A}_p \mathbf{y}_{i,t-p}^* + \mathbf{e}_{it}^* \\ \mathbf{y}_{it}^* &= \mathbf{A} \mathbf{x}_{it}^* + \mathbf{e}_{it}^* \\ i &\in \{1, 2, \dots, N\}, t \in \{1, 2, \dots, T\} \end{aligned} \tag{2.14}$$

where the star symbol (\*) signifies a variable that has transformed. To denote the original variable, the transformation uses the forward orthogonal deviation (FOD) of  $\mathbf{y}_{it-1}^*$  equal to  $c_t[y_{i,t-l} - (y_{i,t-l+1} + \dots + y_{i,T-l})/(T-t)]$  where  $l = 0, 1, \dots, p$  and  $c_t^2 = (T-t)/(T-t+1)$ .

By combining the time unit observations with  $p$  representing the number of parameters, the resulting model is as follows.

$$\mathbf{Y}_i^* = \mathbf{X}_i^* \mathbf{A}' + \mathbf{E}_i^* \quad (2.15)$$

where

$$\begin{aligned} \mathbf{Y}_i^* &= (\mathbf{y}_{i1}^*, \dots, \mathbf{y}_{iT_1}^*)' \\ \mathbf{X}_i^* &= (\mathbf{x}_{i1}^*, \dots, \mathbf{x}_{iT_1}^*)' \\ \mathbf{E}_i^* &= (\mathbf{e}_{i1}^*, \dots, \mathbf{e}_{iT_1}^*)' \\ \mathbf{A} &= (\boldsymbol{\alpha}_1^*, \dots, \boldsymbol{\beta}_p^*) \end{aligned}$$

If presented in vector-matrix form, the model is as follows.

$$\text{vec}(\mathbf{Y}_i^*) = (\mathbf{I}_k \otimes \mathbf{X}_i^*) \text{vec}(\mathbf{A}') + \text{vec}(\mathbf{E}_i^*) \quad (2.16)$$

In parameter estimation using the GMM method, instrument variables  $\mathbf{Q}_i$  that are uncorrelated with each column in the transformed variable matrix  $\mathbf{E}_i^*$  are used. These instrument variables  $\mathbf{Q}_i$  ensure that there is no correlation between the predictors and the error term in the model. The following is the instrument variable matrix (Hayakawa, 2016).

$$\begin{aligned} \mathbf{Q}_i &= \text{diag}(\mathbf{q}_{i1}', \dots, \mathbf{q}_{iT_1}') \\ \mathbf{q}_{it} &= (\mathbf{y}_{i,t-r}', \dots, \mathbf{y}_{i,t-1}')' \text{ and } \mathbf{y}_{it}' = [y_{i,t} \quad x_{i,t}] \\ &\text{with } t = 1, \dots, T_1 \text{ and } 1 \leq r \leq a \end{aligned} \quad (2.17)$$

where the maximum value of  $r$  is defined as  $a \geq 4$ .

The selection of  $a$  is performed by testing each value individually. For example, by testing  $a = 4$ , followed by parameter estimation and instrument validity testing. If the validity test indicates that the instrument is valid, the next step is to conduct a significance test on the parameters.

## 2.7. Identification of the PVAR Model Order

Identifying the order  $p$  in the PVAR model is the process of determining the optimal number of lags to include in the model. The optimal lag is the number of lags that have a significant effect, so it is necessary to examine the data and assess the appropriate lag length. Choosing the right lag is crucial because too few lags may lead to underfitting, while too many lags may result in overfitting. The optimal lag indicates how long it takes for one variable to respond to another and helps eliminate issues of autocorrelation in the PVAR model. Andrews and Lu (2001) proposed the Model and Moment Selection Criteria (MMSC) for the GMM model, which is based on Hansen's (2012) J statistic related to overidentification restrictions. The proposed MMSC aligns with various commonly used likelihood-based model selection criteria, such as the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), and Hannan-Quinn Information Criterion (HQIC), where the optimal lag is chosen based on the smallest value of these criteria (Sihombing et al., 2022). Andrews and Lu recommend using  $MMSC_{BIC}$  or  $MMSC_{HQIC}$ , because  $MMSC_{AIC}$  does not meet the consistency criteria due to its positive probability. The MMSC with the lowest value indicates the optimal lag, while the coefficient of determination captures the proportion of variation explained by the PVAR model at different lags (Sigmund & Ferstl, 2017).

If we apply the MMSC to the GMM estimator, the proposed criterion selects the pair of vectors  $(p, q)$  that minimizes the following:

$$\begin{aligned} MMSC_{BIC,n}(k, p, q) &= J_n(k^2 p, k^2 q) - (|q| - |p|)k^2 \ln n \\ MMSC_{AIC,n}(k, p, q) &= J_n(k^2 p, k^2 q) - 2k^2(|q| - |p|) \\ MMSC_{HQIC,n}(k, p, q) &= J_n(k^2 p, k^2 q) - Rk^2(|q| - |p|) \ln \ln n \quad R > 2 \end{aligned}$$

where

- $J_n$  : The test statistic employed to examine the validity of the instrumental variables, as indicated in Equation (2.19)
- $k$  : The number of variables included in the model
- $p$  : The number of lag orders

- q : The number of moment conditions, which is equivalent to the number of lags
- n : The total number of observations

## 2.8. Parameter Significance Test

The parameter significance test is conducted to assess how significant the effect of the explanatory variables is on the response variable. The effect of each explanatory variable on the response variable is measured using a partial test. In this regard, the  $t_{value}$  formula used is as follows (Sigmund & Ferstl, 2017).

$$t_{value} = \frac{vec(\hat{\Phi})}{Se(vec(\hat{\Phi}))} \quad (2.18)$$

where  $vec(\hat{\Phi})$  is the result of the parameter estimation and  $Se(vec(\hat{\Phi}))$  is the standard error of the estimated parameter value (Hayakawa, 2016).

$$Var(vec(\hat{\Phi})) = \frac{1}{N} [(I_k \otimes S'_{QX}) \hat{\Omega}^{-1} (I_k \otimes S_{QX})]^{-1}$$

$$Se(vec(\hat{\Phi})) = \sqrt{\frac{1}{N} [(I_k \otimes S'_{QX}) \hat{\Omega}^{-1} (I_k \otimes S_{QX})]^{-1}}$$

where  $S_{QX} = \frac{1}{N} \sum_{i=1}^N Q'_i \Delta^* W_{minus,i}$ , and  $\hat{\Omega}^{-1}$ .

The following hypotheses are used to test the significance of the parameters on a partial basis.

$$H_0 : vec(\hat{\Phi}) = \mathbf{0}$$

$$H_1 : vec(\hat{\Phi}) \neq \mathbf{0}$$

The decision criterion is to reject  $H_0$  if the calculated  $t_{value}$  is greater than the critical  $t_{(NT-1)}$  dengan taraf nyata  $\alpha$ . degrees of freedom at a significance level  $\alpha$ . Alternatively, if the p-value is  $\leq \alpha$  then  $H_0$  is rejected, indicating that the explanatory variables have a significant effect on the response variable.

## 2.9. Instrument Variable ( $Q_i$ ) Validity Test

The validity test of instrument variables is conducted to ensure that the instrument variables ( $Q_i$ ) used in the GMM estimation of the PVAR model meet the validity criteria. Validity, in this context, refers to the absence of correlation between the instrument variables ( $Q_i$ ) and the error component. This validity test is performed using the Sargan-Hansen test statistic ( $J_n$ ). The following presents the test statistics.

$$J_n = n\mathbf{m}_n' \hat{\mathbf{\Omega}}^{-1} \mathbf{m}_n \quad (2.19)$$

where

$$\mathbf{m}_n = \frac{1}{n} \sum_{i=1}^n (\mathbf{I}_k \otimes \mathbf{Q}_i)' \text{vec}(\Delta^* \tilde{\mathbf{E}}_{i,t})$$

Where  $\hat{\mathbf{\Omega}}^{-1}$  is the weighting matrix used in the estimation of PVAR parameters,  $Q_i$  is the instrumental variable matrix employed in the GMM estimation of the PVAR model parameters, and  $\text{vec}(\hat{\mathbf{\Phi}})$  is obtained from the first-stage estimation of the PVAR model. Meanwhile,  $(\Delta^* \tilde{\mathbf{E}}_{i,t})$  is a  $(T - 1 - p) \times m$  ordered matrix derived using the following equation:

$$\Delta^* \tilde{\mathbf{E}}_i = \Delta^* \mathbf{W}_i - \Delta^* \mathbf{W}_{\text{minus},i} \hat{\mathbf{\Phi}}$$

where  $\Delta^* \mathbf{W}_i$  represents the current data of the dependent variable (after transformation), and  $\Delta^* \mathbf{W}_{\text{minus},i}$  refers to the lagged data of the dependent variable. The hypotheses used are as follows.

$$H_0 : E \left( (\mathbf{I}_k \otimes \mathbf{Q}_i)' \text{vec}(\Delta^* \tilde{\mathbf{E}}_{i,t}) \right) = 0 \text{ (the instrument is valid.)}$$

$$H_1 : E \left( (\mathbf{I}_k \otimes \mathbf{Q}_i)' \text{vec}(\Delta^* \tilde{\mathbf{E}}_{i,t}) \right) \neq 0 \text{ (the instrument is not valid)}$$

The decision rule is not to reject  $H_0$  if the value of  $J_n$  is less than  $\chi_r^2$  where  $r$  is the number of instrumental variables used minus the number of estimated parameters, or by comparing the  $p$ -value of the test statistic with the significance level  $\alpha$ . If the  $p$ -value  $> \alpha$  then  $H_0$  is not rejected, indicating that the instrumental variables are valid. Conversely, if the  $p$ -value  $< \alpha$  then  $H_0$  is rejected, indicating that the instrumental variables are invalid.

## 2.10. Causality Test of Variables

The Granger Causality Test is a method used to analyze whether a causal relationship exists between two variables. This causality relationship can be either unidirectional or bidirectional. Before conducting the Granger Causality Test, it is crucial to ensure that the data is stationary. If causality exists in economic behavior, the model does not include independent variables, as all variables are treated as dependent variables. The test statistic used is the Wald test statistic for each cross-section unit, with the number of observations being  $T$  periods for each unit.

The Wald test statistic for each cross-section unit, denoted as  $W_{i,T}$ , is given as follows:

$$W_{i,T} = \hat{\theta}_i' \mathbf{R}' (\hat{\sigma}_i^2 \mathbf{R} (\mathbf{Z}_i' \mathbf{Z}_i)^{-1} \mathbf{R}')^{-1} \mathbf{R} \hat{\theta}_i \quad (2.20)$$

where  $\hat{\sigma}_i^2 = (\hat{\mathbf{e}}_i' \hat{\mathbf{e}}_i) / (T - 2p - 1)$ .

In the equation above,  $\hat{\theta}_i$  is the parameter vector estimated for unit  $i$ ,  $\mathbf{R}$  is the restriction matrix,  $\mathbf{Z}_i$  is the matrix of independent variables, and  $\hat{\mathbf{e}}_i$  is the residual vector.

The Granger Causality test statistic for panel data is conducted using the following Wald test.

$$W_{N,T}^{Hnc} = \frac{1}{N} \sum_{i=1}^N W_{i,T} \quad (2.21)$$

The hypotheses used are as follows.

$H_0$  :  $\beta_1 = \beta_2 = \dots = \beta_p = 0$  (there is no effect of X on Y)

$H_1$  : at least one  $\beta \neq 0$  (there is an effect of X on Y)

The decision-making criterion is to reject  $H_0$  if the value of  $W_{N,T}^{Hnc}$  exceeds the critical value of  $\chi_p^2$ , or if the p-value of the test statistic is smaller than the predetermined significance level (Dumitrescu & Hurlin, 2012). This indicates that one variable influences another.

The presence or absence of relationships between variables can be determined by examining the probability values of each causality test and comparing them with significance levels of  $\alpha = 0.05$  (Saskara & Batubara, 2015).

### 2.11. Stability Test of the PVAR Model

The stability test is used to determine whether the constructed PVAR model is stable. If the PVAR model is unstable, the estimation of the Impulse Response Function (IRF) and Variance Decomposition may become invalid. In PVAR, stability is typically assessed by examining the eigenvalues of the coefficient matrix in the estimated VAR system. Each eigenvalue can be either a complex number or a real number. The modulus of an eigenvalue is the distance from that point to the origin in the complex plane. A model is considered stable if all eigenvalues of the coefficient matrix lie within the unit circle, meaning their modulus is less than 1. If any eigenvalue has a modulus of  $\geq 1$ , the model is deemed unstable, requiring modifications such as adjusting the number of lags or changing the estimation method. If an eigenvalue is a real number, then its modulus is equal to its absolute value. If an eigenvalue  $\lambda$  is a complex number of the form  $a + bi$ , then its modulus is calculated as follows.

$$|\lambda| = \sqrt{a^2 + b^2} \quad (2.22)$$

### 2.12. Impulse Response Function (IRF)

The Impulse Response Function (IRF) is used to evaluate the response of endogenous variables over time to shocks in a specific variable, as well as the duration of these shocks. In other words, the IRF aims to assess the impact of a shock in one variable on other variables. This analysis reveals both positive and negative reactions of a variable to changes in another variable. Additionally, the IRF explains the time required for a variable to return to its equilibrium point after experiencing a shock caused by another variable (Amri & Nazamuddin, 2017).

Since the performance of the IRF depends on the underlying estimator, it is crucial to use the GMM estimator with good finite sample properties.

### 2.13. Forecast Error Varians Decomposition (FEVD)

According to Lutkepohl (2005), a closely related tool for interpreting the PVAR model is Forecast Error Variance Decomposition (FEVD). FEVD is used to understand the extent to which a shock in one variable contributes to variations in other variables within a dynamic system. FEVD provides insights into the significance of each variable in explaining the movement of other variables in the PVAR model. This is achieved by decomposing the forecast error into contributions from impulse shocks in the system over several future periods. FEVD in the PVAR model is calculated by decomposing the forecast error variance of a given variable based on the contribution of shocks from each variable in the system. Mathematically, FEVD for variable  $y_i$  at horizon  $h$  is expressed as follows.

$$\omega_{i,j}(h) = \frac{\sum_{k=0}^{h-1} (\Psi_k)_{i,j}^2}{\sum_{k=0}^{h-1} \sum_{j=1}^N (\Psi_k)_{i,j}^2} \quad (2.23)$$

where

$\omega_{i,j}(h)$  : the contribution of variable  $j$  to variable  $i$  at horizon  $h$ .

$\Psi_k$  : the impulse response matrix for lag  $k$ .

$N$  : the number of variables in the PVAR model.

### 2.14. Autocorrelation Test

Autocorrelation is commonly observed in time series data analysis; however, this does not imply that autocorrelation is irrelevant in cross-section data. In cross-section research, testing for autocorrelation is essential to ensure that observations are independent and do not influence one another. Autocorrelation occurs when the residuals from one period are correlated with those from a previous period,

potentially leading to biased and inefficient estimates. One of the most widely used methods for detecting autocorrelation is the Durbin-Watson test. The Durbin-Watson test can be calculated using the following formula:

$$DW = \frac{\sum_{t=2}^n (e_t - e_{t-1})^2}{\sum_{t=1}^n e_t^2}$$

where  $e_t$  is the residual at time  $t$ , and  $n$  is the number of observations.

This test evaluates the presence of autocorrelation by comparing the calculated Durbin-Watson statistic (DW) with the critical values from the Durbin-Watson table, specifically the upper bound ( $dU$ ) and the lower bound ( $dL$ ). The criteria for the test are as follows.

1. If  $DW < dL$ , positive autocorrelation is present.
2. If  $dL \leq DW \leq dU$ , the presence of autocorrelation is uncertain.
3. If  $dU < DW < 4 - dU$ , no autocorrelation is detected.
4. If  $DW > 4 - dL$ , negative autocorrelation is present.
5. If  $4 - dU \leq DW \leq 4 - dL$ , the presence of autocorrelation remains uncertain.

### 2.15. Non-Statistic Review

Gross Regional Domestic Product (GRDP) represents the total market value of goods and services produced within a specific region or province over a given year. GRDP is categorized into two types: GRDP at constant prices and GRDP at current prices (Lestari et al., 2022). GRDP at current prices is used to observe the economic structure, while GRDP at constant prices is used to assess economic growth (Amelia et al., 2023). There are three main factors influencing a country's economic growth: capital accumulation, which includes all forms of new investments in land, physical equipment, capital, or human resources; population growth, which contributes to capital accumulation over the coming years; and advancements in science and technology within the country. Economic growth occurs when a country produces a greater quantity of goods and services, which can be observed through an increase in GRDP value.

The economic growth rate is typically expressed as a percentage and generally has a positive value, indicating economic growth. However, it can also be negative. Negative economic growth occurs when the current national income is lower than that of the previous period. The formula used to calculate the GRDP growth rate is as follows.

$$GRDP \text{ Growth Rate} = \frac{GRDP_n - GRDP_{n-1}}{GRDP_{n-1}} \times 100\% \quad (2.24)$$

Inflation refers to a general increase in the prices of goods and services, which can reduce people's purchasing power. This can trigger economic instability, prompting the government to implement monetary policies to maintain price and inflation stability (Rosnawintang et al., 2020). Inflation is a continuous rise in overall price levels, often caused by a mismatch between economic policies—such as production, money supply, and price regulation—and the income level of the population (Putong, 2016). Inflation is closely related to the Consumer Price Index (CPI), as it directly affects purchasing power (Hasanudin, 2021). A situation in which the value of a currency declines or weakens, accompanied by rising prices, can be considered an inflationary phenomenon (Aryani & Maupula, 2021).

The inflation rate can be defined as the percentage increase in the price of goods over a specific period. This measurement is conducted using the Consumer Price Index (CPI) to analyze changes in the prices of goods and services. When the prices of goods rise, producers tend to increase production, leading to higher income for producers. High and unstable inflation reflects economic instability, which can result in a continuous and widespread increase in the prices of goods and services. This, in turn, contributes to higher poverty rates in Indonesia (Suyanto, 2023). Based on the CPI, the overall rate of price increases can be calculated. The following is the formula for calculating inflation.

$$Inflation = \frac{CPI_n - CPI_{n-1}}{CPI_{n-1}} \times 100\% \quad (2.25)$$

$$Inflation \text{ Rate} = \frac{Inflation_n - Inflation_{n-1}}{Inflation_{n-1}} \times 100\% \quad (2.26)$$

### **III. RESEARCH METHODOLOGY**

#### **3.1. Place and Time of Research**

This research was conducted in the odd semester of the 2024/2025 academic year at the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Lampung University, located at Jalan Prof. Dr. Ir. Soemantri Brojonegoro, Gedong Meneng, Rajabasa District, Bandar Lampung City, Lampung.

#### **3.2. Research Data**

The research data used in this study consists of secondary data obtained from the official websites of the Badan Pusat Statistik (BPS) for each province in Indonesia. The data collected for this research includes the inflation rate and the Gross Regional Domestic Product (GRDP) growth rate of Indonesian provinces from 2015 to 2024. The data used is panel data, which consists of both time series and cross-section units. The time series unit in this study consists of annual observations from 2015 to 2024, while the cross-section unit includes observations from 34 provinces in Indonesia. These provinces are Aceh, North Sumatra, West Sumatra, Riau, Jambi, South Sumatra, Bengkulu, Lampung, Bangka Belitung Islands, Riau Islands, DKI Jakarta, West Java, Central Java, DI Yogyakarta, East Java, Banten, Bali, West Nusa Tenggara, East Nusa Tenggara, West Kalimantan, Central Kalimantan, South Kalimantan, East Kalimantan, North Kalimantan, North Sulawesi, Central Sulawesi, South Sulawesi, Southeast Sulawesi, Gorontalo, West Sulawesi, Maluku, North Maluku, Papua, and West Papua.

### 3.3. Research Methodology

This research was conducted through a literature study in the form of theoretical analysis and computational practice. The computational practice was carried out using Stata software to perform descriptive statistic analysis, data plotting, panel data stationarity testing, order identification, parameter estimation, parameter significance testing, instrument variable validity testing, Granger causality testing, model stability testing, Impulse Response Function (IRF) analysis, and Forecast Error Variance Decomposition (FEVD).

The stages carried out in this research are as follows.

1. Conduct descriptive statistical analysis for each observed dataset and visualize GRDP growth rate and inflation rate through plotting.
2. Perform panel data stationarity testing using the Im, Pesaran-Shin (IPS) test. If the data is not stationary, apply differencing.
3. Identify the order ( $p$ ) for the PVAR model using the MMSC method. The corresponds to the lowest values of  $MMSC_{BIC}$  and  $MMSC_{HQIC}$ .
4. Construct the instrument variable ( $Q_i$ ).
5. Estimate the parameters of the Panel VAR model using the Generalized Method of Moments (GMM).
6. Conduct significance testing for the parameters in the PVAR model.
7. Validate the instrument variables ( $Q_i$ ) using the Sargan-Hansen test.
8. Perform the Granger causality test on panel data to determine the direction of the relationship between the two variables.
9. Test the stability of the PVAR model.
10. Analyze the Impulse Response Function (IRF).
11. Conduct Forecast Error Variance Decomposition (FEVD) analysis.
12. Interpret the PVAR model results.

The steps of the Panel VAR data analysis can be illustrated in a flowchart, as shown in Figure 1.

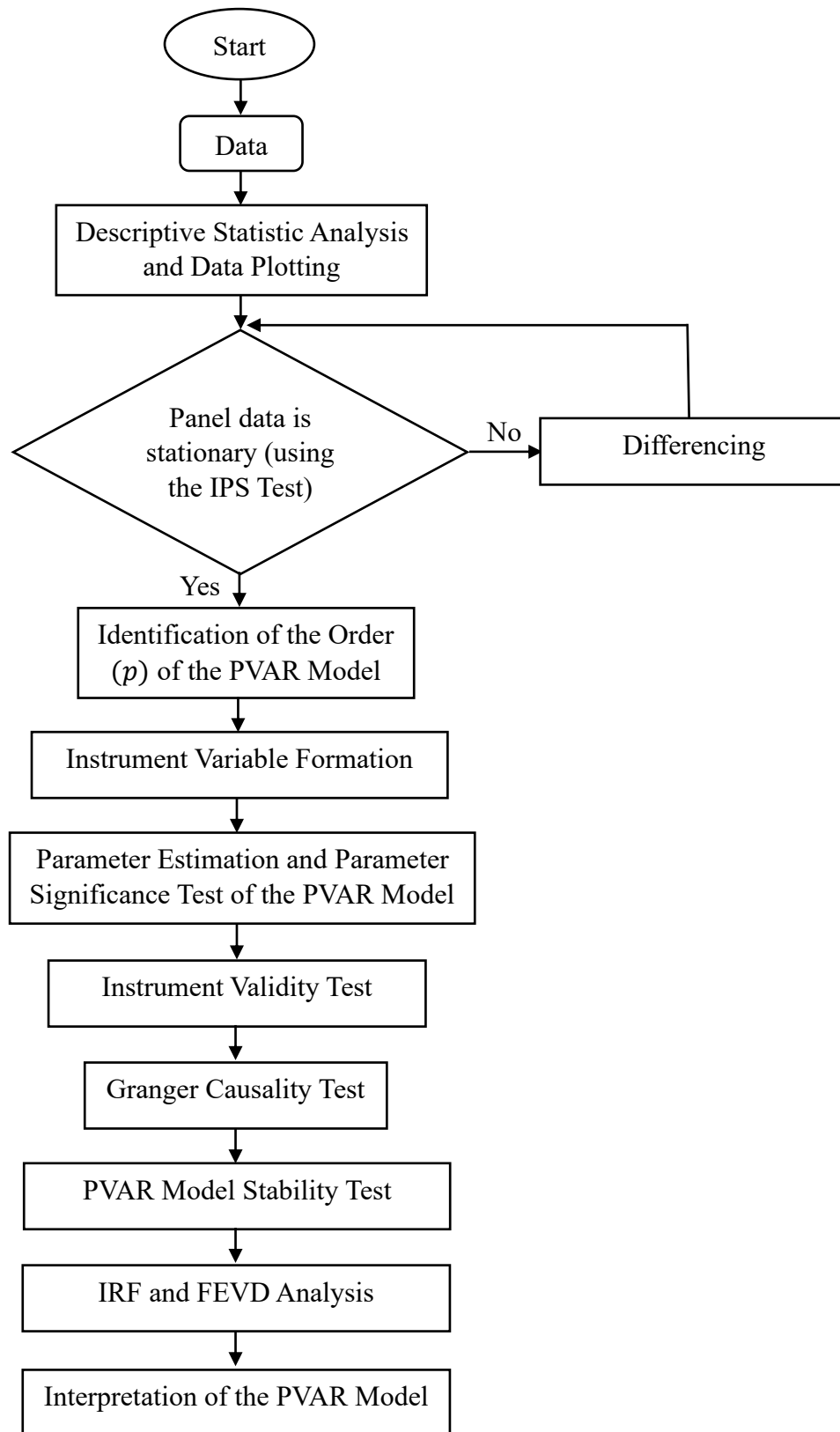


Figure 1. Flowchart of PVAR Analysis.

## V. CONCLUSION

1. Based on the analysis of panel data on GRDP growth rates and inflation rates across 34 provinces in Indonesia from 2015 to 2024, the following conclusions can be drawn there is a bidirectional relationship between GRDP growth and inflation rates, meaning that GRDP growth influences inflation rate movements, and conversely, inflation rate movements also affect GRDP growth. The PVAR model formed for GRDP growth and inflation rate data is as follows:

$$\text{GRDP}_{i,t} = 0.1097 \text{GRDP}_{i,t-1} - 0.3435 \text{Inflation}_{i,t-1} + 0.0118 \text{GRDP}_{i,t-2} \\ - 0.2522 \text{Inflation}_{i,t-2} + e_{i,t}$$

$$\text{Inflation}_{i,t} = 0.3987 \text{GRDP}_{i,t-1} + 0.6487 \text{Inflation}_{i,t-1} - 0.0474 \text{GRDP}_{i,t-2} \\ - 0.3233 \text{Inflation}_{i,t-2} + e_{i,t}$$

2. Based on the estimation results of the PVAR model, there is a dynamic relationship between GRDP growth and inflation, where both variables influence each other in the short and long term. A 1% increase in GRDP growth from the previous year (at  $t - 1$ ) will raise the current year's GRDP growth (at  $t$ ) by 0.1097%. However, it will also increase the current year's inflation rate by 0.3987%. In contrast, if GRDP growth increased two years prior (at  $t - 2$ ) by 1%, the impact on current GRDP growth is relatively small, at only 0.0118%, while its effect on the current inflation rate is negative, reducing inflation by 0.0474%. This indicates that short-term economic growth tends to trigger inflation, but in the long run, its effect diminishes and may even slightly suppress inflation. On the other hand, high inflation in the previous year (at  $t - 1$ ) by 1% will reduce the current year's GRDP growth by 0.3435% while simultaneously increasing the current inflation rate by 0.6487%. This effect suggests that high inflation tends to be persistent and can hinder economic

growth. Inflation from two years prior (at  $t - 2$ ) also negatively impacts current economic growth, where a 1% increase in past inflation will reduce the current year's GRDP growth by 0.2522% and decrease the current inflation rate by 0.3233%.

3. This finding indicates that although past inflation can negatively affect economic growth, in the long run, economic adjustment mechanisms may help suppress the inflation rate itself. Overall, the relationship between economic growth and inflation is bidirectional, where economic growth tends to drive inflation, while high inflation can act as a barrier to economic growth.

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