

ABSTRACT

ROLLE'S THEOREM AND THE MEAN VALUE THEOREM FOR FUNCTIONS TAKING VALUES IN THE SEQUENCE SPACE ℓ_1

By

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Rolle's Theorem and the Mean Value Theorem are studied for sequence-valued functions in the Banach space ℓ_1 , which is the space of sequences equipped with the norm

$$\|p\|_1 = \sum_{d=1}^{\infty} |p_d|.$$

The space ℓ_1 allows the extension of fundamental concepts of real analysis, such as limits, continuity, and derivatives, to infinite-dimensional spaces. The discussion is carried out using a mathematical analysis approach based on literature study, employing the definitions of continuity, differentiability, and the completeness property of the space ℓ_1 .

The results show that Rolle's Theorem and the Mean Value Theorem remain valid for sequence-valued functions in the space ℓ_1 , provided that the function is continuous on a closed interval and differentiable on an open interval. The proof is obtained by examining the derivatives of each sequence component and utilizing the properties of the ℓ_1 norm, ensuring the existence of a point within the interval that satisfies the conditions of both theorems. These results confirm that fundamental theorems of differential analysis on $\mathbb{R}$ can be generalized to Banach spaces, particularly the space ℓ_1 .

Keywords: Rolle's Theorem, Mean Value Theorem, ℓ_1 space, Banach space, sequence-valued functions, derivative.

ABSTRAK

TEOREMA ROLLE DAN TEOREMA NILAI RATA-RATA PADA FUNGSI BERNILAI BARISAN ℓ_1

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Teorema Rolle dan Teorema Nilai Rata-Rata dibahas pada fungsi bernilai barisan dalam ruang Banach ℓ_1 , yaitu ruang barisan yang memiliki norma

$$\|p\|_1 = \sum_{d=1}^{\infty} |p_d|.$$

Ruang ℓ_1 memungkinkan pengembangan konsep dasar analisis real, seperti limit, kekontinuan, dan turunan, ke ruang berdimensi tak hingga. Pembahasan dilakukan melalui pendekatan analisis matematis berbasis studi literatur dengan memanfaatkan definisi fungsi kontinu, fungsi terdiferensialkan, serta sifat kelengkapan ruang ℓ_1 .

Hasil pembahasan menunjukkan bahwa Teorema Rolle dan Teorema Nilai Rata-Rata tetap berlaku pada fungsi bernilai barisan di ruang ℓ_1 apabila fungsi kontinu pada interval tertutup dan terdiferensialkan pada interval terbuka. Pembuktian diperoleh dengan meninjau turunan setiap komponen barisan serta menggunakan sifat norma pada ruang ℓ_1 , sehingga diperoleh suatu titik pada interval yang memenuhi kondisi kedua teorema tersebut. Hasil ini menegaskan bahwa teorema dasar analisis diferensial pada $\mathbb{R}$ dapat diperluas ke ruang Banach, khususnya ruang ℓ_1 .

Kata-kata kunci: Teorema Rolle, Teorema Nilai Rata-Rata, ruang ℓ_1 , ruang Banach, fungsi bernilai barisan, turunan.